

A REAL-TIME BUS DISPATCHING POLICY TO MINIMIZE HEADWAY VARIANCE

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Simon J Berrebi

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Approved by:

Dr Kari Watkins, Advisor
School of Civil and Environmental
Engineering
Georgia Institute of Technology

Dr Jorge Laval
School of Civil and Environmental
Engineering
Georgia Institute of Technology

Dr Michael Rodgers
School of Civil and Environmental
Engineering
Georgia Institute of Technology

Dr Alan Erera
H. Milton Stewart School of Industrial
and Systems Engineering
Georgia Institute of Technology

Dr. Juan Carlos Muñoz
Department of Transportation
Engineering and Logistics
Pontificia Universidad Catlica de Chile

Date Approved: 7 April 2014

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ERRATA

The following is an errata for page 26, equations (41) and (42).

Equations (41) and (42) in the proof of Theorem 3.3.5 say that if:

$$P(D_j^i = 1) = P\left(j = \arg \max_r \left[\sum_{q=i}^{i+r} \frac{K_q + s_i}{r+1} \right]\right) \quad (1)$$

Then

$$\sum_{j=i^o+1}^n P(D_j^{i^o+1} = 1) \left[\frac{\sum_{q=i^o+1}^j E[k_q | D_j^{i^o+1} = 1, k_2, \dots, k_i]}{j - (i^o + 1)} \right] = E \left[\max_r \frac{\sum_{q=i^o+2}^{i^o+1+r} K_q}{r} \right] \quad (2)$$

However, there the term s_i is omitted in the expectation term. The equality in Equation (41) in Theorem 3.3.5 should be an approximate equality. Indeed, it would be exact if the following error terms were all equal to zero.

$$\epsilon'_j = E \left[\frac{\sum_{q=i^o+1}^r K_q + s_{i^o}}{r - i^o} | D_j^{i^o} = 1 \right] - E \left[\frac{\sum_{q=i^o+1}^r K_q}{r - i^o - 1} | D_j^{i^o} = 1 \right] = 0 \quad \forall j \quad (3)$$

Tracing back the ϵ'_j terms to the error on the hold a_i^* , denoted ϵ , we get that:

$$\epsilon \leq \frac{\sum E[I_j^i] \epsilon_j}{1 + \frac{\sum E[I_j^i]}{j-i}} \leq \max_j \epsilon'_j \quad (4)$$

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	iii
LIST OF TABLES	vi
LIST OF TABLES	vi
LIST OF FIGURES	vii
LIST OF FIGURES	vii
SUMMARY	viii
I INTRODUCTION	1
II LITERATURE REVIEW	4
2.1 Causes and Consequences of Bus Bunching	4
2.2 Automatically Collected Data	6
2.3 Bus Travel Time Predictions	6
2.3.1 Static Predictions	7
2.3.2 Real-time predictions	8
2.4 Bus Dispatching Policies	10
III STOCHASTIC DECISION PROCESS	16
3.1 Introduction	16
3.2 Formulation	17
3.2.1 State and Action Spaces	17
3.2.2 Transition Probabilities	18
3.2.3 Total Expected Cost Criterion	19
3.3 Structural Properties of an Optimal Policy	21
3.4 Discussion	27
IV SIMULATION	29
4.1 Model Structure	29
4.2 Control Policies	30

4.2.1	The Headway-Based Policy	30
4.2.2	The Self-Coordinating Policy	31
4.2.3	The Proposed Dispatching Policy	32
4.3	Results	32
4.3.1	Instability	33
4.3.2	Perturbation	36
V	CONCLUSION	38
	Bibliography	40

LIST OF TABLES

1	Summary of real-time prediction methods in the literature.	9
2	Summary of variable definitions.	17

LIST OF FIGURES

1	Self-coordinating dispatching policy	14
2	State and action at decision epoch i	18
3	State, action and inter-arrival time at decision epoch 1	19
4	Passenger waiting time versus level of systemic instability	34
5	Mean headway versus level of systemic instability	34
6	Headway coefficient of variation versus level of systemic instability . .	35
7	Mean passenger waiting time versus number of dispatches	36

SUMMARY

Transit agencies include buffer time in their schedules to maintain stable headways and avoid bus bunching. In this work, a real-time holding mechanism is proposed to dispatch buses on a loop-shaped route, solely based on operating conditions in real-time. Holds are applied at the terminal station to minimize the expected variance of bus headways at departure. The bus-dispatching problem is formulated as a stochastic decision process. The optimality equations are derived and structural properties of the optimal policy are inferred by backward induction. The exact optimal holding policy is then found in closed form, as a function of the expected travel time of buses currently running. A simulation assuming stochastic operating conditions and unstable headway dynamics is performed to assess the expected average waiting time of passengers at stations. The proposed control strategy is found to provide lower passenger waiting time and better resiliency than methods recommended in the literature and used in practice.

CHAPTER I

INTRODUCTION

Frequency and reliability are critical elements of bus quality of service. For passengers, frequency is freedom because it determines the waiting components of a trip [1]. A frequent service makes good use of passengers' time; it allows them to decide when they want to travel and to change their travel plans. When bus routes operate at high frequency, passengers don't need to plan their trips around a schedule. With headways of up to 10-15 minutes, passengers tend to arrive randomly and to wait for the next passing bus [2]. Headway regularity is important because randomly arriving passengers have proportionally more chance of arriving during a long headway than during a short one. Headway regularity defines the reliability of a bus route and the amount of trust that passengers can have in it [6].

Bus bunching is the natural tendency for buses to catch up to each other along the route and to eventually travel as a platoon. Bus bunching is a source of concern for both transit riders and providers. It increases passenger waiting time and crowding by augmenting the variability of headways. When buses are traveling together, they operate at the speed of the leading vehicle, which is the slowest. Bunching also forces passengers to arrive early at stations and to budget long travel time [3]. For transit agencies, bunching restricts the use of scarce resources. Slow operating speeds mean low frequency and capacity. Several bunched vehicles have the utility of a single long bus. On a platoon of vehicles, several bus drivers perform a task that could be done by a single worker. Additionally, due to the random nature of the headway dynamics, bus bunching makes it difficult for operations' planners to schedule vehicles and operators.

In practice and in the literature, the increasingly reliable real-time data on bus operating conditions (location, load, traffic conditions etc.) are being disseminated to passengers through applications on mobile devices [4]. With these information, passengers can decide how, when, and where to travel at the last minute. On the other hand, transit operations are usually planned well in advance and real-time decision making is rarely established as a systematic procedure. Bus bunching occurs because buses are dispatched with headways too short to return to the terminal station before their time of next dispatch [5]. To avoid bunching, planners insert slack time in pre-timed operations, reducing the capacity of service. In this work, a simple control strategy is found to dispatch buses using real-time arrival predictions to maintain stable headways and avoid bus bunching.

The subject of study is a loop shaped route with n buses. Vehicles perform a trip on the route then come back to the control point: the terminal station. The headway of a bus is the time since the last passing of a vehicle at its current location. To prevent bus bunching before its formation, it is important to keep bus headway variance as low as possible. In this work, a control strategy is proposed to minimize expected headway variance at the terminal station. The bus dispatching problem is solved as a finite horizon stochastic decision process because decisions to hold buses are made sequentially, in a system that evolves randomly as a function of travel times. Operationally, it functions as a regular Computer Aided Dispatching system (CAD) in which drivers are told how long to dwell upon their arrival at the terminal station.

In Chapter 2, a literature review on bus bunching and on the available methods to avoid it is presented. Section 2.1 lays out the literature on the nature and the causes of bus bunching. Sections 2.2-2.4 provide a review of the literature in the control process involving data, travel time predictions and dispatching policies. Section 2.2 gives an over-view of the available data recording technologies to serve in the modeling process of bus trajectories. Using data from Section 2.2, the forecasting methods presented

in Section 2.3 infer the future states of the system from past behaviors. Based on predicted outcomes of immediate decisions, the dispatching policies in Section 2.4 aim to maintain the system in a stable state.

In Chapter 3, the bus dispatching problem is formally addressed as a stochastic decision problem. In Section 3.1, the problem is formulated. Action and State spaces are defined along with transition probabilities and the expected variance criterion. In Section 3.2, the optimality equations are derived and structural properties of an optimal dispatching policy are investigated. Using backward induction, an exact optimal solution is found in its closed form as a function of the joint-probability distribution of bus arrival-times at the terminal station. A short discussion is given in Section 3.3.

In Chapter 4, the proposed control policy is compared with control methods recommended in the literature and used in practice through discrete-event simulation. The simulation re-creates a typical loop-shaped route with unstable and random headway dynamics. In Section 4.1, the model structure is described. In Section 4.2, the mechanism of the control policies under comparison is explained and results are given in Section 4.3.

CHAPTER II

LITERATURE REVIEW

The bus dispatching problem was first addressed in the literature in the 1960's. Since then, it has received surges of attention each time new methods and technologies became available. This chapter surveys the literature on bus bunching itself and on the methods to avoid it. Section 2.1 outlines the causes and consequences of bus bunching identified in the literature. In the context of transit operations, a control policy is a systematic decision rule whose aim is to yield a specific system behavior. A predictive model is necessary to establish the dependency between the control and the evolution of the system. The predictive model is a representation of the future as a function of the past. To analyse the available information to a decision maker, Section 2.2 surveys available data sources and Section 2.3 lays out the static and real-time bus travel time prediction methods used in practice and recommended in the literature. Based on these predictions, the dispatching policies in section 2.4 seek to send buses with regular headways at high frequency.

2.1 Causes and Consequences of Bus Bunching

The Transit Capacity and Quality of Service Manual (TCQSM) considers that bus bunching is mainly a passenger crowding problem [6]. In general, the amount of delay that a bus encounters along the route is subject to the trajectories and loads of downstream buses. Running time is composed of link travel time, during which buses drive from one station to the next, and dwell time, during which buses board and alight passengers. When a bus arrives at a station with a long headway, it must board a relatively greater number of passengers and thereby accumulate further delay. These passengers will also have to alight, making it even more difficult for the

vehicle to recover. When, on the other hand, a bus arrives at a station with a short headway, correspondingly fewer passengers have time to arrive and the bus dwells for a relatively short time. Strathman, et al. conducted a study on bus trajectories and headway dynamics using automatically collected information [7]. Automatic Vehicle Location (AVL) and Automatic Passenger Count (APC) data suggested that bus bunching resulted from a failure to maintain headways. As the disparity between headways grew, leading buses were slowed down by dwelling operations while trailing buses had few passengers to board and to alight, forcing the buses to bunch into a single platoon. The degree of instability of a bus route depends a plethora of factors including payment method, frequency of stops, share of *atypical passengers* etc. [8] When a bus route is in equilibrium, ie. buses are evenly spaced and the operating conditions are stationary, even the slightest perturbation such as traffic lights can destabilize the system [6].

The TCRP Report 135 , Controlling System Costs, gives a step-by-step methodology for creating bus schedules. The concept of running time is defined by its role in pre-timed operations and by its stochastic nature.

The impact of running times can be a significant factor in unreliable operations, particularly with regard to bunching of vehicles. While bunching is often caused by varying operating conditions (traffic, congestion, loading variability, etc.) the scheduled run times can affect whether vehicles operate early or late. And once vehicles fall outside the schedule, this affects loadings and can cause bunching [5].

Schedules or any type of dispatching policy should be able to dispatch vehicles on time: to let large gaps in time form between departures is to induce bus bunching from the start. Furthermore, according to the Synthesis of Transit Practice 15, when a bus departs late from the terminal station, then it is unlikely that it will recover its lost time [9]. On both headway-based and schedule-based routes, a bus *operating*

late arrives at the control point too long after the last dispatch occurred. When this occurs, the lagging vehicle is usually sent immediately on its route but its headway at departure is longer than the others.

In the following sections, a literature review for the three main components of a bus dispatching policy is presented. Since the deactivation of GPS selective availability in 2000, the quality and cost of automatically collected data has largely improved. This new source of data has allowed researchers and agencies to model bus headway dynamics and predict bus arrival times both on the planning horizon and in real time. Based on these predictions, much of the research in the literature has focused on bus real-time control. These developments are described in Sections 2.2, 2.3 and 2.4.

2.2 Automatically Collected Data

In Supervision Strategies for Improved Reliability of Bus Routes, the results of a 1989 survey among twenty transit agencies throughout the United-States and Canada were reported [9]. The survey asked respondents about their methods to monitor ridership and travel times. None of the participating transit agencies used automatic data collection to monitor these statistics. In 2013, the American Public Transportation Association conducted a similar study, surveying Seventy-five transit agencies members of APTA [10]. More than 70% of transit agencies reported that they were able to track buses in real-time. Of the agencies that had real-time location technology, the vast majority could track more than 90% of their fleet. Of the agencies that didn't have access to these technologies, 92% were interested in adopting them.

2.3 Bus Travel Time Predictions

Bus travel time prediction methods can be separated into two groups. Static predictions are made at the planning horizon typically weeks or months in advance of publishing a schedule. They are made by schedulers without specific information on the operating conditions. Real-time predictions are based on automatically collected

information about real-time operating conditions and are updated often. Chapter 3 will assume the availability of a real-time prediction algorithm giving an unbiased joint probability distribution of bus arrival-times.

2.3.1 Static Predictions

The predictions used for scheduling at the planning horizon, typically weeks or months in advance, use historical data to infer the running times. Schedulers do not have information about the specific operating conditions at the time a bus is to be dispatched. Histograms are usually used as the predictive model for the probability distribution of bus running times. According to a survey conducted for the TCRP Report 135, 80% of the participating agencies use historic running times as their prediction for running time, typically disregarding extreme values [5]. This method is used along with professional judgment, to make sure that buses effectively return to the terminal station before their time of next dispatch. The underlying assumption of non-real-time prediction is that buses will face the same route characteristics they have in the past [11]. A potential limitation of this approach is that the running time of subsequent buses are not independent. Bus trajectories influence each other through the unstable headway dynamics that cause bus bunching. The running times of successive bus assignments may contain more variability than the sample variance obtained from the histograms.

To contend with the variability of running time and to prevent it from disturbing pre-timed operations, the TCRP Report 113 recommends using percentiles of historical running time as the prediction [12]. This method comes from the Synthesis of Transit Practice 15 [9], which lays out best practices for scheduling and controlling buses. The synthesis argues that time points along a route should be set with tight thresholds to avoid operators having to *kill time*. The allocated running time (essentially the prediction used for bus dispatching at the terminal) should, however

correspond to the 95th percentile of observed values to avoid vehicles starting their route late. This method can only be applied if the quality and quantity of available historical data is sufficient. Automatically collected data generally allow agencies to use these more sophisticated metrics.

2.3.2 Real-time predictions

There are a number of alternatives to using historical data as probability distributions. The running time of a bus is a random variable and its probability distribution is correlated with random events that occur before and during its run. The literature includes multiple methods to assess the impact of operating conditions on running time. Table 1 presents a summary of the literature in this field. Input variables are the independent factors that were found to have an effect on running time. Inference method is the technique used to derive the prediction from historic and current data. A forecasting model that is self-adjustable in real time is capable of updating the prediction with changing operating conditions. Models that consider dwell time separately can include the behavior of buses at stations and therefore consider bus bunching. Methods that have measures of uncertainty allow users to build confidence intervals by providing the standard deviation of their prediction. Table 1 summarizes the factors considered and methodologies of the prediction algorithms.

Some authors have focused on evaluating the relative impact of different factors on running time to revise predictions as these factors change. Abdelfattah and Khan developed a simulation of a bus route to test running time with respect to various properties of the route [13]. Their research concluded that traffic density in veh/lane/km has a cubic relation with deviations from schedules. Patnaik and collaborators used a forecasting model that considers dwell time separately in the prediction [11]. They evaluated expected delay caused by cumulative dwell time on a route segment using linear regression. Mazloumi and collaborators calculated running time probability

Table 1: Summary of real-time prediction methods in the literature.

Reference	Input Variables	Inference method	Self-Adjustable in Real-Time	Seperate Consideration of Dwell Time	Measure of Uncertainty
Abdelattah and Khan (1998)	Route Length Number of Stations Number of Buses Bus Efficient Ratio Number of Stops Traffic Density	Regression	NO	NO	NO
Cathey and Dailey (2003)	Real-Time Positioning Traffic Conditions Weather	Linear Interpolation	YES	NO	NO
Jeong et Al. (2004)	Real-Time Positioning Dwell Time	Neural Network	YES	YES	NO
Patnaik et Al. (2004)	Route Length Number of Stations Dwell Time Boardings and Alightings Weather	Linear Regression	NO	YES	NO
Shalaby and Farhan (2004)	Real-Time Positioning Link Running Time Dwell Time	Kalman Filter	YES	YES	YES
Sun et Al. (2007)	Real-Time Positioning	Linear Interpolation	YES	NO	NO
Mazloumi et Al. (2010)	Route Length Number of Stations Number of Signalized Intersections Delay	Regression	NO	NO	YES
Mazloumi et Al. (2011)	Real-Time Positioning Traffic Conditions Weather	Neural Network	YES	NO	YES

distributions by departure time windows [14]. They found that the probability distribution for running time is generally symmetric for peak-hour departures and skewed for off-peak departures.

Forecasting models have also been developed with the ability to update their predictions in real-time, mainly for the purpose of disseminating real-time information to passengers. Cathey and Dailey created a prediction algorithm for expected deviations from schedule at each station [15]. Traffic conditions and weather were modeled into a discrete set. For each operating condition, the prediction was a function of bus time-space coordinates. Jeong and Rilett used a Neural Network to predict running times [16]. A Neural Network is a model capable of teaching itself the relation between variables and to predict their future values based on historical and current information. In [16], cumulative dwell time on route segments was fed into the model. The Neural Network then evaluated the expected delays resulting from excess dwell

time. Shalaby and Farhan developed a model that incorporated the notion of headway in its prediction for dwell time [17]. A Kalman Filter was used to estimate the passenger arrival rates and to predict the impact of lateness on dwell time. Sun et al. modeled bus running time as a finite state machine [18]. The method was capable of quickly detecting unexpected incidents and to update the prediction accordingly. Mazloumi et al. developed a Neural Network with traffic conditions and schedule adherence as input parameters [19]. Along with the expected running time, the Neural Network was capable of yielding the standard deviation of the prediction and allowed to construct confidence intervals.

Finally, hickman developed a control strategy that will be detailed in Section 2.4 [20]. The research was based on a stochastic headway dynamics model to predict running time and passenger waiting time at downstream stations. This model assumed that the route is headway-based with randomly arriving passengers. It accounted for the covariance between headways and loads. The model was an extension of [21] to allow for real-time updated predictions. Although the prediction was never empirically validated, its main assumptions are relatively simple and could potentially serve as a basis for a prediction algorithm.

2.4 Bus Dispatching Policies

A typical approach for bus scheduling is to decide on a policy headway through service standards. These standards impose headways short enough to satisfy required levels of capacity and of frequency [22]. Then, according to the TCRP Report 30 Transit Scheduling, the required number of vehicles is determined as:

$$\frac{\text{Cycle Time}}{\text{Desired Headway}} \quad (1)$$

Where cycle time is running time plus recovery time [23]. The TCRP Report 135 recommends to discretize the day into departure time windows and to maintain

a common headway during each period to make a good use of buses and drivers [5]. Even though the running time prediction may change within a departure time window, a unique value is kept. This method is analogous to defining headways as a function of predicted cycle time and the number of available buses.

Control strategies to mitigate bus bunching emerged before the development of automatically collected information. Dispatching buses on a loop route with a single control point was addressed by Osuna and Newell in 1972[24]. The running times were independent, identically distributed, and headway dynamics were not considered. The problem was formulated as a infinite horizon Markov Decision Process (MDP). The objective was to minimize total expected passenger wait time at stops, which is a linear function of headway variance. The state space consisted of the time since last departure from the terminal station of every vehicle on the route. An approximate optimal policy was found for up to two vehicles. In follow-on work, the effects of headway lengths on dwell time were considered, as was the natural tendency of buses to bunch [24]. The objective of the research was the same as in [24] and an approximate optimal dispatching policy was found. The research recommended to apply enough control to keep the effects of bunching *"well under control, perhaps even so as to have a negligible effect on the average wait"*. The control schemes, however are not applicable because in both models the dynamics become intractable when several vehicles and stations are introduced. Until recently, the pure real-time dispatching problem introduced by Osuna and Newell has not been re-addressed. Subsequent literature focused on maintaining on-route stable operations, assuming just-in-time dispatches from the terminal station.

Barnett also addressed the problem of minimizing expected average waiting time, but on a route where buses begin their assignments on time [26]. Bus bunching occurs along the route as a random process resulting from unstable headway dynamics. Control is applied locally to avoid bus bunching, and an approximate optimal control

policy was found.

Since the end of selective availability of GPS in 2000, control strategies supported by real-time information have also emerged. The research has mainly focused on developing a model for headway dynamics and minimizing average expected waiting time in stationary operating conditions. Using slack time built in the schedules at every time-point, the following control strategies aim to maintain schedules or planned headways, which are based on predictions.

Eberlein and Wilson developed heuristic methods to avoid light-rail bunching in a deterministic environment [27]. The vehicles are dispatched on time from the terminal station and the headways are controlled at several points along the route to avoid the perturbation of headway dynamics. The control strategy in [20] was a line search on the holding time to minimize passenger waiting. The control method solves for one hold at a time and so the expression for headways in the future does not consider subsequent holding decisions. The control algorithm makes immediate holding decisions based on the predicted resulting waiting time. In [28], the objective was to minimize expected average passenger waiting time at stations neighboring the control points. Decisions were based on perfect predictions and bus bunching was modeled as a deterministic process. Delgado and collaborators formulated the bunching problem as a total waiting time minimization problem considering boarded and waiting at-stop passengers [29]. The modeled dynamics were deterministic and all the control measures were determined at once, supported by a perfect prediction. Buses could be held but also refuse passengers to increase their operating speed. Capacity constraints were considered without using binary variables. An exact optimal solution was found by branch-and-bound.

Van Oort and collaborators developed a holding strategy that could be applied on both headway-based and schedule-based bus routes [30]. On headway-based routes, the method consisted in holding buses that arrived at time-points with short headways

for a fraction of their deviation from the planned headway. On schedule-based routes the control was similar with time of arrival instead of planned headway. Daganzo defined a dimensionless parameter β as the marginal increase in expected bus delay arising from a unit increase in headway [31]. Bus holding at a station was a linear function of β times the deviation from the target headway. Xuan and collaborators took the same approach with an additional term ϵ : the deviation from schedule [32]. The holding mechanism was found to minimize the required slack for a given level of schedule reliability. In [31] and [32], it was demonstrated that the headway variance was a bounded function of deviations from schedule, assuming stationary operating conditions. Aside from the β parameter, these models didn't require predictions for running time because their objective was to maintain pre-timed operations using slack time built in the schedules at every station.

Using real-time information, Bartholdi and Eisenstein addressed the vehicle dispatching problem of Osuna and Newell. They found a powerful heuristic to send buses with low mean and low variance. Without concern for a schedule or planned headway, the control mechanism aimed to to recover the *natural headway* [33]. When a bus arrives at the control point, it is held for a fraction of the predicted time until the next arrival, k_2 . The hold a_1 applied on a bus is the following:

$$a_1 = \alpha k_2 \tag{2}$$

Accordingly, each bus is dispatched with a headway equal to the average of two inter-arrival times:

$$\alpha k_2 + (1 - \alpha)k_1 \tag{3}$$

A bus in between two short headways (k_1 and k_2) will be dispatched with a short headway. When bunching occurs, a long headway precedes a series of very short ones. The control mechanism applies significant control only on the last headway in

structural properties of an optimal dispatching policy. The predicted travel times will be considered as input parameters of the control policy.

CHAPTER III

STOCHASTIC DECISION PROCESS

3.1 Introduction

The variance of bus headways is both a cause and a consequence of bus bunching. A negative correlation exists between headways of adjacent vehicles, therefore so the variance of headways tends to grow along a route [20]. The expected passenger wait time at stops is a linear function of headway variance [24]. To avoid bus bunching and undue passenger waiting, it is imperative that buses begin their route with stable headways. In this problem, an agent can hold vehicles when they arrive at the terminal station and then re-dispatch them onto the route. The objective of this work is to find a control policy that minimizes the expected variance of headways at departure.

There are n buses on the route and the agent has an unbiased joint probability distribution for the arrival time of all $n - 1$ buses currently running. We say the problem is an n -epoch decision process because the expected variance criterion is evaluated with respect to n headways. Each time a bus arrives at the terminal station, the agent must decide how long it should be held, so as to minimize the expected variance of the next n headways at departure. This performance measure is also a function of future decisions. The optimal decision rules for future holds will be determined by backward induction as functions of the outcome of random variables. Then, assuming optimal future decisions, an immediate decision will be found as part of an optimal policy.

In this chapter, bus dispatching is addressed as a finite-horizon stochastic decision process. In Section 3.2 the problem is formulated, state and actions spaces are described along with the transition probabilities and the cost function. In Section

3.3, an optimal bus dispatching policy is found. The expected variance criterion is expressed recursively with respect to bus dispatches. Convexity of the expected variance is demonstrated and the policy yielding minimal expected variance is found exactly in its analytical form by recursion. A short discussion follows in Section 3.4.

Table 2 summarizes the definitions of this section.

Variable	Table 2: Summary of variable definitions. Definition
n	Number of buses on the route
a_i	Hold applied on the i^{th} passing bus
S_i	State of the system at epoch i (Random Variable)
s_i	State of the system at epoch i (Outcome)
K_i	Time between the arrival of the i^{th} and the $i + 1^{th}$ bus (Random Variable)
k_i	Time between the arrival of the i^{th} and the $i + 1^{th}$ bus (Outcome)
π	Dispatching policy
$u_i^\pi(a_{i-1}^o, k_2, \dots, k_i)$	Total expected cost from epoch i on, assuming a_i, k_2, \dots, k_i
π^*	Optimal dispatching policy

3.2 Formulation

3.2.1 State and Action Spaces

To define the bus dispatching problem, we will use the notation introduced in Chapter 1 and Markov decision theory [34]. When the i^{th} bus arrives at the terminal station, we say that the system enters the i^{th} decision epoch, and that the system state is $s_i \in \mathbb{R}$. The system state s_i is the time between the departure of bus_{i-1} and the arrival of bus_i at the terminal station (see Figure 2). At each decision epoch i in $\{2, \dots, n - 1\}$, an action $a_i \in [[-s_i]^+, \infty)$ ¹ corresponds to a decision to hold the i^{th} bus for a_i time units. The lower bound on a_i assures that $s_i + a_i \geq 0$ for reasons that will soon be discussed, and that $a_i \geq 0$, which is consistent with the notion that holds can only be applied in the forward time direction, ie. the agent can never delay a bus for a negative amount of time. By convention, the action at the last decision epoch $a_n = 0$.

¹Let by notation $[x]^+ = \max[x, 0]$

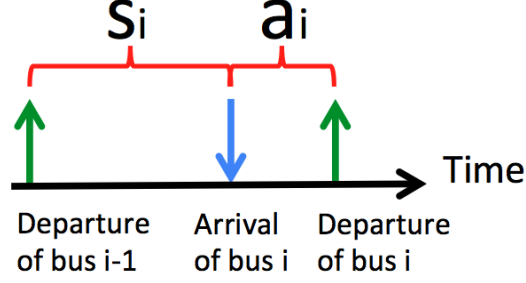


Figure 2: State and action at decision epoch i

3.2.2 Transition Probabilities

The system state at future decision epochs will depend on the arrival time of buses currently running. In Figure 3, their probability distributions are represented by blue normal curves. Let the random variable K_i denote the inter-arrival time of bus_i i.e. the time between the arrival of bus_{i-1} and bus_i at the terminal station. The outcome of K_i is k_i . At the beginning of the problem, when the bus at the terminal station is bus_1 , the joint probability distribution for the inter-arrival times of each bus currently running is $f(k_2, \dots, k_n)$. Because random headway dynamics have been modeled in different manners, we will keep $f(k_2, \dots, k_n)$ in its general form. Any prediction described in the literature review, or future prediction algorithms yet to be developed, can be inputted in place of $f(k_2, \dots, k_n)$. Note that the K_i 's are not necessarily independent. Firstly, the travel time of buses currently running will be commonly affected by random operating conditions such as traffic, rain, accidents etc. Secondly, by the unstable headway dynamics described in [20], bus trajectories are correlated with each other.

The state of the system at a future epoch i , S_i is the time between the departure of bus_{i-1} and the arrival of bus_i at the terminal station. The state at epoch i is the inter-arrival time less the hold applied on the last passing bus:

$$S_i = K_i - a_{i-1} \quad (5)$$

Given s_{i-1} and a_i , the random variable S_i is a function of K_i . Note that the random

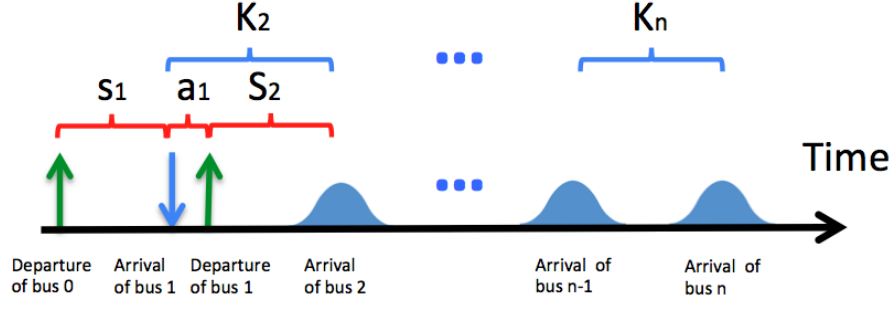


Figure 3: State, action and inter-arrival time at decision epoch 1

variables, K_i , are independent of the actions because holds take place at the terminal station, which is upstream of the route portions that currently running vehicles have to cover. The random variables, K_i , however are history dependent so the dispatching problem is not a Markov Decision Process. The outcome of the state at the i^{th} decision epoch is:

$$s_i = k_i - a_{i-1} \quad (6)$$

At each decision epoch, the headway of the departing bus is $s_i + a_i$. In Figure 2, the inter-departure time of bus_1 is the space between the first and the second green upward arrow. Passing is not allowed so each headway at departure should be positive, ie. $s_i + a_i \geq 0$ for all i .

3.2.3 Total Expected Cost Criterion

Our ultimate objective will be to construct a policy that will minimize headway variance such that the bus being controlled will not have to be held when it returns to the control point. Since the cycle time of the bus at the terminal station is random, we treat the sample headway variance for any dispatching policy as a random variable. By the Koenig-Huygens theorem we have that the statistical variance in headways at departure can be expressed as follows:

$$Var = \sum_{i=1}^n (S_i + a_1)^2 - n \left[\frac{\sum_{i=1}^n (S_i + a_i)}{n} \right]^2 \quad (7)$$

$$= \sum_{i=1}^n (K_i - a_{i-1} + a_1)^2 - \frac{[s_1 + \sum_{i=2}^n (K_i)]^2}{n} \quad (8)$$

$$E[Var] = E \left[\sum_{i=1}^n (K_i - a_{i-1} + a_1)^2 \right] - E \left[\frac{[s_1 + \sum_{i=2}^n (K_i)]^2}{n} \right] \quad (9)$$

The objective is to minimize the expected variance of headways at departure. Note that the term $E[\frac{[s_1 + \sum_{i=2}^n (K_i)]^2}{n}]$ is independent of the actions. It therefore suffices to minimize the first term of (9) with respect to the actions to minimize the expected variance. This term will be expressed as a cost criterion. A policy π is a systematic decision rule. At each decision epoch, a policy dictates an action as a function of past states and decisions. At each decision epoch, an expected total cost criterion is u_i^π if policy π is applied going forward in time. The expected total cost criterion corresponds to the sum of the expected headways at departure squared for each bus that has not yet been controlled. This criterion can be expressed iteratively as in (10) and expended as in (11) and (12) by the Law of Iterated Expectations.

$$u_i^\pi(a_{i-1}^o, k_2, \dots, k_i) = (s_i + a_i^\pi)^2 + E[u_{i+1}^\pi(a_i, k_2, \dots, k_i, K_{i+1})] \quad (10)$$

$$= (s_i + a_i^\pi)^2 + \sum_{j=i+1}^n E[(K_j - a_{j-1}^\pi + a_j^\pi)^2 | k_2, \dots, k_i] \quad (11)$$

$$= (s_i + a_i^\pi)^2 + \sum_{j=i+1}^n \int_0^\infty \dots \int_0^\infty f(k_2, \dots, k_i, k_{i+1}, \dots, k_n) (k_j - a_{j-1}^\pi + a_j^\pi)^2 \delta k_{i+1} \dots \delta k_n \quad (12)$$

In Equation (12), a_j^π for $j > i$ will depend on action and state history past epoch i . Since state transitions are random, the future actions picked by a policy are random. A policy π^* is said to be optimal if:

$$u_1^{\pi^*}(s_1) \leq u_1^\pi(s_1) \forall \pi \quad (13)$$

The objective of this work is to find the optimal dispatching policy π^* , and most importantly, the optimal immediate action $a_1 \in \pi^*$.

3.3 Structural Properties of an Optimal Policy

In this section, Bellman's principle of optimality is applied to separate the sequential dispatching problem into n overlapping sub-problems. The i^{th} sub-problem consists in finding the optimal policy starting at a decision epoch i as a function of $\{a_{i-1}, k_2, \dots, k_i\}$. Lemma 3.3.1 shows that a holding sub-policy $\pi'_i = \{a'_i, \dots, a'_{n-1}\}$ that minimizes the total expected cost criterion starting at epoch i is part of the optimal policy.

Lemma 3.3.1. *Given a_{i-1}^o and $\{k_2, \dots, k_i\}$, for any i , the action a_i^* is defined as follows:*

$$a_i^* = \text{ArgMin}_{a_i} [(k_i - a_{i-1}^o + a_i)^2 + \sum_{j=i+1}^n E[(K_j - a_{j-1}^* + a_j^*)^2 | k_2, \dots, k_i]] \quad (14)$$

At each decision epoch, $\{a_i^*, \dots, a_{n-1}^*\}$ minimizes $u_i^\pi(s_i, k_2, \dots, k_i)$.

Proof. The statement is true for $i = n - 1$ because for any $\{k_2, \dots, k_{n-1}\}$ and a_{n-2}^o , by definition of a_{n-1}^* , we have that for any arbitrary policy $\pi'_{n-1} = \{a_{n-1}\}$:

$$(k_{n-1} - a_{n-2}^o + a_{n-1}^*)^2 + E[(K_n - a_{n-1}^*)^2 | k_1, \dots, k_{n-1}] \leq u_{n-1}^\pi(k_{n-1} - a_{n-2}^o, k_2, \dots, k_{n-1}) \quad (15)$$

Suppose that $\{a_{i+1}^*, \dots, a_{n-1}^*\}$ minimizes $u_{i+1}^\pi(s_{i+1}, k_2, \dots, k_i)$, then given $\{k_2, \dots, k_i\}$ and a_{i-1}^o we have for any arbitrary policy $\pi = \{a_i, \dots, a_{n-1}\}$:

$$(k_i - a_{i-1}^o + a_i^*)^2 + \sum_{j=i+1}^n E[(K_j - a_{j-1}^* + a_j^*)^2 | k_1, \dots, k_i] \quad (16)$$

$$\leq (k_i - a_{i-1}^o + a_i^\pi)^2 + E[(K_{i+1} - a_i^\pi + a_{i+1}^*)^2] + \sum_{j=i+2}^n E[(K_j - a_{j-1}^* + a_j^*)^2 | k_1, \dots, k_i] \quad (17)$$

$$\leq (k_i - a_{i-1}^o + a_i^\pi)^2 + \sum_{j=i+1}^n E[(K_j - a_{j-1}^\pi + a_j^\pi)^2 | k_1, \dots, k_i] \quad (18)$$

$$= u_i^\pi(a_i, k_2, \dots, k_i) \quad (19)$$

By backward induction, the optimal policy is $\{a_1^*, \dots, a_{n-1}^*\}$ and $\pi_i'^* = \{a_i^*, \dots, a_{n-1}^*\} \in \pi^*$. \square

At each decision epoch, the policy $\pi_i'^*$ minimizes $u_i^\pi(a_{i-1}^o, k_2, \dots, k_i)$ for a given $a_{i-1} \notin \pi_i'$. When a real bus arrives at the control and it becomes bus_1 , then a dispatching policy $\{a_1^*, \dots, a_{n-1}^*\}$ is optimal because the hold a_0 cannot be affected by the current policy. Any sub-policy that does not include a_1 should be viewed as part of a sub-problem that needs to be solved in order to determine the optimal holding policy and immediate action.

If the problem was not in continuous-time, π^* could be found by dynamic programming. This method consists in evaluating $u_i^\pi(a_{i-1}^o, k_2, \dots, k_i)$ as a function of a_i for every possible combination of $\{a_{i-1}^o, k_2, \dots, k_i\}$. The a_i that minimizes the total expected cost would then be retained and the total expected cost would be re-evaluated at $i - 1$. The algorithm would converge when the iteration reaches the first decision epoch. This method however could not be applied to this problem without discretizing $f(k_2, \dots, k_n)$ because there exists an infinite number of instances for any K_i .

For the remainder of this chapter, further structural properties of the optimal policy are investigated. Lemma 3.3.2 demonstrates that $u_i^\pi(a_{i-1}^o, k_2, \dots, k_i)$ is convex in $\{a_i, \dots, a_{n-1}\}$. This result will later be used to derive the optimal dispatching policy.

Lemma 3.3.2. *The total expected cost criterion is convex in the action space.*

Proof. Suppose that the total expected cost criterion at the first decision epoch is not convex in the action space. Then there exists two action vectors, $\bar{a}' = [a'_1, \dots, a'_{n_1}]^T$ and $\bar{a}'' = [a''_1, \dots, a''_{n_1}]^T$, and a $t \in [0, 1]$ such that:

$$u_1^{t\bar{a}' + (1-t)\bar{a}''} \geq tu_1^{\bar{a}'} + (1-t)u_1^{\bar{a}''} \quad (20)$$

Expanding the total cost criterion as per (12) gives:

$$\sum_{i=1}^n \int_0^\infty \dots \int_0^\infty f(k_1, \dots, f_n) [k_i + t(-a'_{i-1} + a'_i) + (1-t)(a''_{i-1} + a''_i)]^2 \delta k_1 \dots \delta k_n \quad (21)$$

$$\geq t \sum_{i=1}^n \int_0^\infty \dots \int_0^\infty f(k_1, \dots, f_n) [k_i - a'_{i-1} + a'_i]^2 \delta k_1 \dots \delta k_n \quad (22)$$

$$+ (1-t) \sum_{i=1}^n \int_0^\infty \dots \int_0^\infty f(k_1, \dots, f_n) [k_i - a''_{i-1} + a''_i]^2 \delta k_1 \dots \delta k_n \quad (23)$$

$$= \sum_{i=1}^n \int_0^\infty \dots \int_0^\infty f(k_1, \dots, f_n) [t[k_i - a'_{i-1} + a'_i]^2 + (1-t)[k_i - a''_{i-1} + a''_i]^2] \delta k_1 \dots \delta k_n \quad (24)$$

Noticing that the k_i term is equal in (21) and (24) for any i and $f(k_1, \dots, k_n)$, we have a contradiction by the Cauchy inequality. As a result there exists no \bar{a}' and \bar{a}'' such that (20) holds. Therefore u_1 is convex and by extension u_i is convex for any i . \square

In this work, holding is the only available form of control. Vehicles may be delayed but never advanced, therefore holds are always non-negative. The feasible region of a policy π is:

$$a_1 \geq 0 \quad (25)$$

$$\dots \quad (26)$$

$$a_{n-1} \geq 0 \quad (27)$$

Clearly, the feasible region is also convex. Therefore any inflexion point in the total cost criterion inside the feasible region is a global minimum [35]. If there exists no inflexion point inside the feasible region, then the closest boundary point to an inflexion point is a global minimum. In the following Corollary, a_i^* is derived as a function of the future departure headways and their derivative with respect to the decision i .

Corollary 3.3.3. *Given a_{i-1}^o and $\{k_2, \dots, k_i\}$, the optimal action a_i^* as defined in*

Lemma (3.3.1) is equal to:

$$a_i^* = [a_{i-1}^o - k_i - \sum_{j=i+1}^n E[\frac{\delta[-a_{j-1}^* + a_j^*]}{\delta a_i} [k_j - a_{j+1}^* + a_j^*] | k_2, \dots, k_i]]^+ \quad (28)$$

Proof. The derivative of $(k_i - a_{i-1}^o + a_i)^2 + \sum_{j=i+1}^n E[(K_j - a_{j-1}^* + a_j^*)^2 | k_2, \dots, k_i]$ with respect to a_i is:

$$2(k_i - a_{i-1}^o + a_i) + 2 \sum_{j=i+1}^n E[\frac{\delta[-a_{j-1}^* + a_j^*]}{\delta a_i} [k_j - a_{j+1}^* + a_j^*] | k_2, \dots, k_i] \quad (29)$$

If there exists an a_i such that (29) equal zero, then it is a_i^* . If there exists no such a_i , then by convexity of $u_i^\pi(a_i, k_2, \dots, k_i)$, $a_i^* = 0$. \square

The main difficulty in evaluating (28) comes from the non-linearity of the term $\frac{\delta a_j^*}{\delta a_i^*}$. Due to the constraint $a_j \geq 0$, a_j^* is not continuously differentiable at 0 with respect to a_i . Whether or not a_j^* will be positive for a given action a_{i-1} and history $\{k_2, \dots, k_i\}$ depends on the outcome of the random variables $\{K_{i+1}, \dots, K_j\}$. The expectation in (28) is evaluated with respect to every possible combination of inter-arrival times, including all those that would yield a null derivative and all those that would not yield a null derivative. So far, no assumption was made on the the joint probability distributions of the inter-arrival times $f(k_2, \dots, k_n)$. To maintain the generality of the dispatching problem, let us define the following indicator random variable:

Definition.

$$D_j^i = \begin{cases} 1 & \text{if } a_q^* > 0 \ \forall q \in \{i+1, \dots, j-1\} \text{ and } a_j^* = 0 \\ 0 & \text{otherwise} \end{cases} \quad (30)$$

At the i^{th} decision epoch, $P(D_j^i = 1)$ is the probability that the outcome of $\{K_{i+1}, \dots, K_j\}$ will be such that the first bus to leave the terminal station uncontrolled will be bus_j . Note that $P(D_{j_1}^i = D_{j_2}^i = 1) = 0$ for any $j_1 \neq j_2$. The following theorem is central to the construction of an optimal policy for the bus dispatching problem. The indicator random variables D_j^i are used to separate (28) in piecewise linear terms.

In Theorem 3.3.4, the optimal holding policy is found in terms of $f(k_1, \dots, k_n)$ and $P(D_j^i = 1)$.

Theorem 3.3.4. *At each decision epoch i , given a_{i-1} and $\{k_2, \dots, k_i\}$, the optimal holding policy is:*

$$a_i^* = \left[\frac{\sum_{j=i+1}^n P(D_j^i = 1) \left[\frac{\sum_{q=i}^j E[k_q | D_j^i=1, k_2, \dots, k_i]}{j-i} \right] - k_i + a_{i-1}^o}{1 + \sum_{j=i+1}^n \frac{P(D_j^i=1)}{(j-i)}} \right]^+ \quad (31)$$

Proof. By definition $a_n = 0$. Therefore $P(D_{n-1}^n = 1) = 1$. The lhs of (31) for $i = n - 1$ is:

$$\frac{E[k_n | k_2, \dots, k_{n-1}] - k_{n-1} + a_{n-2}^o}{2} \quad (32)$$

This result is consistent with equation (28) as $\frac{\delta a_{n-1}}{\delta a_{n-1}} = 1$ and $\frac{\delta a_n^*}{\delta a_{n-1}} = 0$.

Suppose Equality (31) is true for every $q > i$, then for any $q^o > i$ we have:

$$\frac{\delta a_{q^o}^*}{\delta a_i} = \frac{1}{1 + \sum_{j=i+1}^n \frac{P(D_j^{q^o}=1)}{(j^o - q^o)}} \frac{\delta a_{q^o-1}^*}{\delta a_i} \quad (33)$$

Given that $D_{j^o}^i = 1$, we obtain:

$$\left[-\frac{\delta a_{q^o-1}^*}{\delta a_i} + \frac{\delta a_{q^o}^*}{\delta a_i} \right] = \left[-1 + \frac{1}{1 + \frac{1}{j^o - q}} \right] \frac{\delta a_{q^o-1}^*}{\delta a_i} = \frac{-1}{j^o - q^o + 1} \frac{\delta a_{q^o-1}^*}{\delta a_i} \quad (34)$$

By recursion we get:

$$\frac{\delta a_{q^o-1}^*}{\delta a_i} = \prod_{s=1}^{q^o-i} \frac{1}{1 + \frac{1}{j^o - q^o + s}} = \frac{j^o - q^o + 1}{j^o - i + 1} \quad (35)$$

Combining Equation (34) and (35), we obtain:

$$\left[-\frac{\delta a_{q^o-1}^*}{\delta a_i} + \frac{\delta a_{q^o}^*}{\delta a_i} \right] = \frac{-1}{j^o - i + 1} \quad (36)$$

By Corollary 3.3.3, it is known that:

$$a_i^* = [a_{i-1}^o - k_i - \sum_{j=i+1}^n E[\frac{\delta[-a_{j-1}^* + a_j^*]}{\delta a_i} [k_j - a_{j+1}^* + a_j^*] | k_2, \dots, k_i]]^+ \quad (37)$$

$$= [a_{i-1}^o - k_i - \sum_{j=i+1}^n P(D_j^i = 1) \sum_{j=i+1}^n E[\frac{\delta[-a_{j-1}^* + a_j^*]}{\delta a_i} [k_j - a_{j+1}^* + a_j^*] | k_2, \dots, k_i, D_j^i = 1]]^+ \quad (38)$$

$$= [a_{i-1}^o - k_i - \sum_{j=i+1}^n P(D_j^i = 1) \sum_{j=i+1}^n E[\frac{-1}{j-i+1} [k_j - a_{j+1}^* + a_j^*] | k_2, \dots, k_i, D_j^i = 1]]^+ \quad (39)$$

$$= \left[\frac{\sum_{j=i+1}^n P(D_j^i = 1) \left[\frac{\sum_{q=i}^j E[k_q | D_j^i = 1, k_2, \dots, k_i]}{j-i} \right] - k_i + a_{i-1}^o}{1 + \sum_{j=i+1}^n \frac{P(D_j^i = 1)}{(j-i)}} \right]^+ \quad (40)$$

The induction hypothesis is validated. \square

From Theorem 3.3.4 we have the optimal dispatching policy as a function of $f(k_1, \dots, k_n)$ and D_j^i . The final step of the derivation of an optimal dispatching policy will consist in expressing $P(D_j^i = 1)$ exactly as a function of $f(k_1, \dots, k_n)$.

Theorem 3.3.5. *The probability distribution of D_j^i is the following:*

$$P(D_j^i = 1) = P \left(j = \arg \max_r \left[\sum_{q=i}^{i+r} \frac{K_q + s_i}{r+1} \right] \right) \quad (41)$$

Proof. Once again, we use the backward induction hypothesis. The hypothesis holds trivially for $i = n-1$ and $j = n$. Suppose that the hypothesis holds for all $j > i^o + 1$.

Then for any j satisfying the condition, by the law of iterated expectations, we have:

$$\sum_{j=i^o+1}^n P(D_j^{i^o+1} = 1) \left[\frac{\sum_{q=i^o+1}^j E[k_q | D_j^{i^o+1} = 1, k_2, \dots, k_i]}{j - (i^o + 1)} \right] = E \left[\max_r \frac{\sum_{q=i^o+2}^{i^o+1+r} K_q}{r} \right] \quad (42)$$

Then for i^o and $j > i^o$, $P(D_j^{i^o} = 1)$ can be expressed as follows:

$$P(D_j^{i^o} = 1) = P \left(j = \arg \max_r \left[\sum_{q=i^o+1}^{i^o+r} \frac{K_q + s_{i^o}}{r} \right] \mid D_{i^o+1}^{i^o} \neq 1 \right) P(D_{i^o+1}^{i^o} \neq 1) + P(D_{i^o+1}^{i^o} = 1) \quad (43)$$

By Theorem 3.3.4, the event that $D_{i^o+1}^{i^o} = 1$ is equivalent to:

$$\sum_{j=i^o+1}^n P(D_j^{i^o+1} = 1) \left[\frac{\sum_{q=i^o+1}^j E[k_q | D_j^{i^o+1} = 1, k_2, \dots, k_i]}{j - (i^o + 1)} \right] \leq k_{i^o+1} - a_{i^o}^o \quad (44)$$

$$\Rightarrow E \left[\max_r \frac{\sum_{q=i^o+1}^{i^o+2+r} K_q}{r} \right] \leq k_{i^o+1} - a_{i^o}^o \quad (45)$$

$$\Rightarrow P(D_j^{i^o} = 1) = P \left(j = \arg \max_r \left[\sum_{q=i^o+1}^{i^o+r} \frac{K_q + s_{i^o}}{r - i^o + 1} \right] \right) \quad (46)$$

□

Corollary 3.3.6. *At each decision epoch i , given a_{i-1} and $\{k_2, \dots, k_i\}$, the optimal holding policy is:*

$$a_i^* = \left[\frac{E \left[\max_r \frac{\sum_{q=i^o+1}^{i^o+r} K_q}{r} \right] - s_i}{E \left[1 + \frac{1}{\arg \max_r \left[\sum_{q=i^o+1}^{i^o+r} \frac{K_q}{r} \right]} \right]} \right]^+ \quad (47)$$

3.4 Discussion

In this chapter, the problem of real-time bus dispatching was formulated as a finite horizon decision problem. When a bus arrives at the control point, then it becomes bus_1 and it should be held for a_1^* time units. When the next bus arrives, the agent obtains the probability distribution of the arrival time of bus_1 . The newly arrived vehicle should then become bus_1 and the policy should be re-evaluated with the updated information. This way, it is guaranteed that the agent makes optimal decisions with all available information.

We have shown that a_i^* in Equation (47) is part of an optimal dispatching policy for any i . This result is intuitive; it corresponds to holding each bus until the arrival of the vehicle running the latest. If for example, it is known with probability 1 that $\arg \max_r \left[\sum_{q=i^o+1}^{i^o+r} \frac{K_q}{r} \right] = r^o$, then the optimal policy consists in holding each bus until its headway reaches a fraction of the expected time until arrival of bus_{r^o} .

Accordingly, each bus between i and r^o will be sent with the same expected headway, thereby minimizing the variance and avoiding the formation of bus bunching from the start. Note that the determination of r is only required because of the constraint that $a_i \geq 0$ for all i . Otherwise, the dispatching policy minimizing the first term of the expected variance in (9), would be to hold and accelerate buses so as to dispatch them all with the same headway at departure.

CHAPTER IV

SIMULATION

In this chapter, the proposed dispatching policy is compared through simulation with methods used in practice and recommended in the literature. A typical loop-shaped bus route is re-created. Buses perform routes, then come back to the terminal station and are re-dispatched according to the policy evaluated. The purpose of these simulations is to compare the quality of service and resilience of each dispatching policy on a route with unstable and random headway dynamics.

4.1 Model Structure

The route is composed of 25 stations, upon which 7 buses run continually in discrete time. At the start of the simulation, buses are empty of passengers and separated by three or four stations. When the simulation begins, the arrival time of each bus to the next station is generated. The simulation ends when every bus has been dispatched four times. The fourth run is used for the analysis.

The simulation is discrete-time, with two types of events, arrival at a station and departure from a station. Each time a bus arrives at or departs from a station, the next event is randomly generated. At the end of each event, the simulation clock is advanced to the next scheduled event. The link travel time from one station to the next is independent and identically distributed for every bus on every pair of station. The travel time probability distribution is normal with mean one minute and standard deviation six seconds. The dwell time at each station accounts for the boardings and alighting operations a vehicle must perform.

Passengers arrive at stations randomly and independently of each other according to a Poisson Processes. At each station, there is a random stream of arriving

passengers for each possible destination. When passengers arrive at a station, they board the next passing bus, independently of their destination. Similarly, when a bus stops at a station, all passengers that have arrived to their destination alight the vehicle. The arrival rates of passengers boarding at one station and alighting at another are specified by an origin-destination matrix. Because no passenger may stay in a bus past the terminal station, the probability that a passenger's destination station precedes its origin station is null. The same rate of arrival was assigned to each feasible origin-destination pair. In other words, a boarding passenger has an equal probability to alight at any following station, and an alighting passenger has an equal probability to have boarded at any preceding station.

In this simulation, control may only be applied at the terminal station. When a vehicle finishes its route, it is dispatched according to the policy evaluated. There is no minimum dwell time. In some instances buses may be dispatched immediately at their time of arrival. The three dispatching policies that were tested in this simulation are the Headway-Based policy, the Self-Coordinating policy [33], and the proposed control strategy. These control mechanisms are described in section 4.2 along with the method to simulate their action.

4.2 Control Policies

4.2.1 The Headway-Based Policy

The Headway-Based bus dispatching policy is a control method that aims to dispatch buses with a pre-determined headway without concern for a fixed schedule. The threshold, denoted β is usually determined as a high percentile of cycle time, divided by the number of buses n . Buses that arrive at the control point with a headway shorter than the threshold are held until their headway at departure reaches the threshold. Buses that arrive with headways longer than the threshold are dispatched immediately. The hold imposed on each bus is:

$$a_1 = [\beta - s_1]^+ \quad (48)$$

Headway-Based policies are used to dispatch buses and Bus Rapid Transit (BRT) on many routes around the world. Jarret Walker argues in Human Transit that on high frequency routes, Headway-Based dispatching policies are more effective at providing quality and capacity of service [1]. In general, headway-based policies are more resilient than schedule-based because a threshold headway is maintained so that buses don't start their routes already bunched together.

4.2.2 The Self-Coordinating Policy

The Self-Coordinating policy was proposed by Bartholdi and Eisenstein and is described in further details in section 2.4. Each time a bus arrives at the control point it is either held for a constant α , multiplied by the expected time until next arrival, or until its headway reaches the threshold β . In this simulations, the constant α is set to $\frac{1}{2}$ as per the numerical example in [33]. Each time a bus arrives at the control point, it is held for a_1 time units.

$$a_1 = \max \left[\frac{1}{2} E[K_2], \beta - s_1 \right] \quad (49)$$

Where K_2 is a random variable for the time to arrival of the following bus. An embedded simulation was developed to serve as a high quality unbiased prediction. When a bus arrives at the control point, the embedded simulation is run fifty times and the mean value of K_2 is used as the prediction.

4.2.3 The Proposed Dispatching Policy

The proposed dispatching policy is the control mechanism derived in Chapter 3. Each time a bus arrives at the terminal station, it is held for a_1^* time units.

$$a_1^* = \left[\frac{E \left[\max_r \frac{\sum_{q=2}^{r+1} K_q}{r} \right] - s_1}{E \left[1 + \frac{1}{\arg \max_r \left[\sum_{q=2}^{r+1} \frac{K_q}{r} \right]} \right]} \right]^+ \quad (50)$$

Recall that s_1 is the time since last departure when the bus arrives at the terminal station and that K_i denotes the random inter-arrival time of the i^{th} bus. To find a_1^* , the probability distribution of $\max_r \frac{\sum_{q=2}^{r+1} K_q}{r}$ and $\arg \max_r \left[\sum_{q=2}^{r+1} \frac{K_q}{r} \right]$ is needed. Each time a bus arrives at the control point, the same embedded simulation used for the Self-Coordinating method is run and histograms of the two random variables are generated. The mean of $\max_r \frac{\sum_{q=2}^{r+1} K_q}{r}$ and $1 + \left[\arg \max_r \left[\sum_{q=2}^{r+1} \frac{K_q}{r} \right] \right]^{-1}$ are then used to compute a_i^* .

In section 4.3, the methods to compare the performance of each dispatching policy are explained and results are given.

4.3 Results

The expected average waiting time at the first station was used as the main policy evaluation criterion. As mentioned in Section 3.1, for a fixed mean headway, this criterion is a linear function of headway variance. Since headway variance tends to grow along a route, mean passenger waiting time at the first station is characteristic of the overall system performance [20]. The mean waiting time at the first stations as derived in [24] is then:

$$\frac{\sum_{i=1}^7 (s_i + a_i)^2}{\sum_{i=1}^7 (s_i + a_i)} \quad (51)$$

Where $s_i + a_i$ is the headway at departure of bus_i on its fourth round. This criterion rewards both short and stable headways. Because mean passenger waiting time is a

function of both mean headway and coefficient of variation, these two metrics were also recorded as part of the simulation experiment. The performance of the dispatching policies was evaluated in two simulation experiments. In the first one, presented in Section 4.3.1, the control variable is a dimensionless parameter for systemic instability. In the second experiment, presented in Section 2.3.2, a perturbation is activated after some time.

4.3.1 Instability

As mentioned in the literature review, boarding and alighting operations are the central source of headway instability. Buses running slow will face an increasing number of passengers waiting at stops. The boarding and alighting of these passengers will cause further delay. According to the model presented in [20], the negative correlation of subsequent headways is a monotone increasing function of the time per boarding and alighting operation and of passenger arrival rates. In other words, the longer it takes passengers to board and alight vehicles, the more prone to bus bunching will be the route. In this experiment, we defined a dimensionless instability parameter equal to passenger rate of arrival, multiplied by boarding and alighting time. We found that for a fixed value of the instability parameter, any combination of its components yielded the same passenger waiting time, mean headway and coefficient of variation.

In this experiment, the proposed control strategy was used to determine the thresholds for the Headway-Based and the Self-Coordinating methods. The simulation was run using the 50th and the 85th percentiles of cycle times for the threshold. Figure 4 shows 95th percentile confidence intervals for the mean passenger waiting time in minutes after 20 vehicles dispatches have occurred.

For low levels of systemic instability, the performance of each dispatching method is roughly equal. As the system becomes more unstable, mean passenger waiting

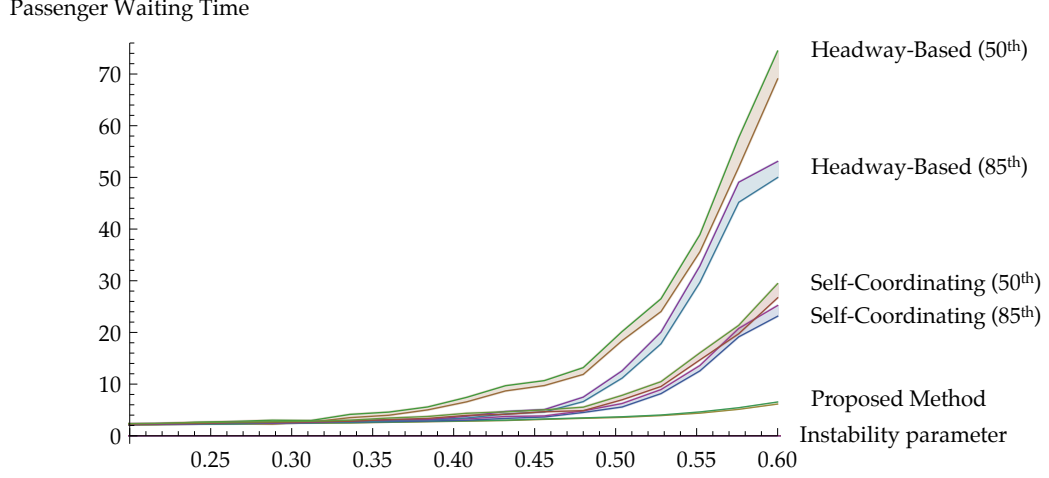


Figure 4: Passenger waiting time versus level of systemic instability

time rises at a high rate with the Headway-Based methods. The Headway-Based method based on the 85th of cycle time maintains lower passenger waiting time than its counter-part based on the 50th percentile. The Self-Coordinating methods perform better than the Headway-Based methods for instable operating conditions, but passenger waiting starts to increase when the dimensionless parameter reaches 0.5. The proposed control method is able to maintain low passenger waiting time for all levels of systemic instability. The following figure shows mean headway in minutes for each control method.

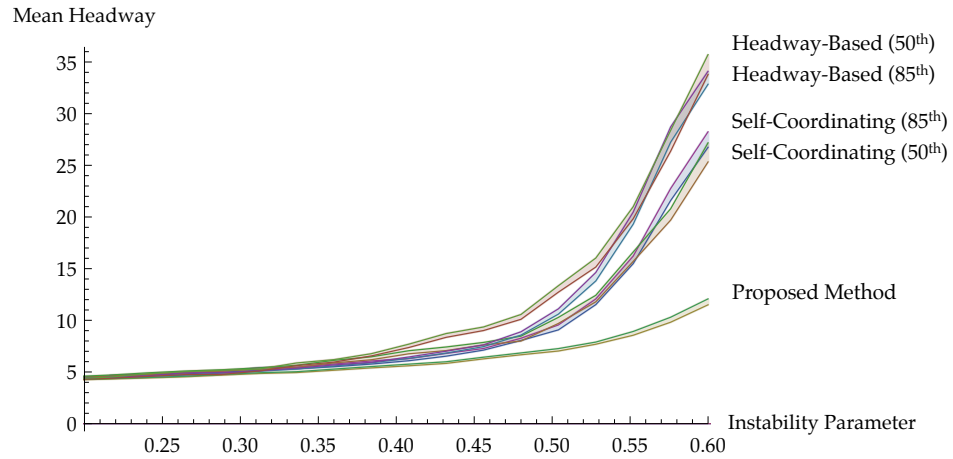


Figure 5: Mean headway versus level of systemic instability

For the Headway-Based and the Self-Coordinating methods, whether the threshold is based on the 50th or the 85th percentile of running time does not greatly affect mean headways. The Self-Coordinating methods produce lower headways than the Headway-Based method. The proposed control strategy maintains the lowest headways throughout, and is moderately affected by the level of instability. The ability of the control methods to keep passenger waiting time and mean headway low is mainly determined by their capacity to control big gaps, and to dispatch buses with stable headways. Figure 6 shows the coefficient of variation of headways at the first station as a function of systemic instability.

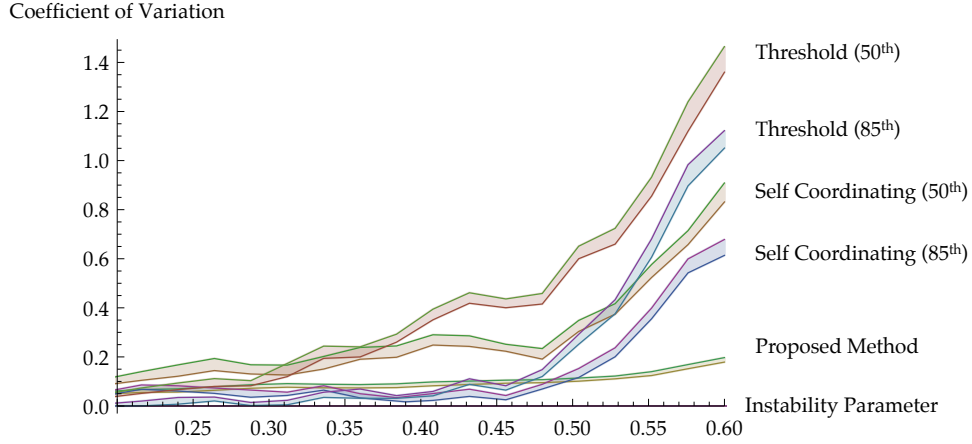


Figure 6: Headway coefficient of variation versus level of systemic instability

The Headway-Based and Self-Coordinating methods based on the 85th percentile of running time have a small coefficient of variation on stable operating conditions. As the dimensionless instability parameter rises, the variance rapidly increases with respect to mean headway for the Headway-Based and Self-Coordinating dispatching policies. For both methods, the increase in passenger waiting time as a function of systemic instability comes with an increase in the coefficient of variation of the same magnitude. The proposed control method maintains the coefficient of variation under 0.2. The slight increase in passenger waiting time of Figure 4 on highly unstable operating conditions can be attributed to higher mean headways. The proposed

control method minimizes headway variance, while maintaining n vehicle departures per cycle time. It is therefore able to adapt to instability by increasing all headways and avoiding the formation of big gaps.

For each value of the dimensionless instability parameter, the simulation was run under stationary operating conditions. In the following section, the experiment involves a perturbation that changes the level on instability.

4.3.2 Perturbation

In this experiment, the route started in stationary operating conditions with a rate of passenger arrivals of 0.36 per minute for each origin-destination stream and one second per boarding and alighting (dimensionless instability parameter of 0.36). When the 70th bus is dispatched, the rate of arriving passengers doubles until that bus returns to the terminal station (seven dispatches later), then the system returns to its initial level of instability. This experiment was conducted to evaluate the ability of dispatching methods to restore equilibrated operations. Figure 7 shows mean passenger waiting time as vehicles are dispatched. The Headway-Based and Self-Coordinating methods were simulated using the 85th percentile of running time of the proposed control method.

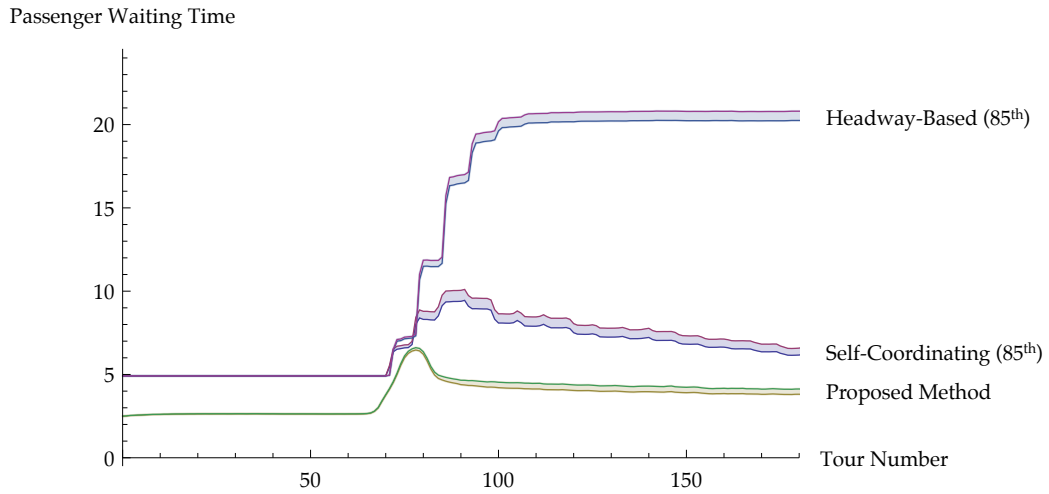


Figure 7: Mean passenger waiting time versus number of dispatches

Before the perturbation is introduced, the Headway-Based and the Self-Coordinating methods systematically dispatch vehicles with headways equal to the relatively high threshold. The mean passenger waiting time under the proposed control method is shorter because buses are dispatched with the lowest possible headways. The Self-Coordinating and the proposed method forecast the perturbation right before it occurs, and are able to adjust their operations accordingly. The Self-Coordinating method holds the last vehicle before the perturbation, while the proposed control method starts increasing headways seven dispatches ahead. When the perturbation is introduced, the Headway-Based method undergoes severe disruptions and passenger waiting time increases steadily, even after the end of the perturbation, to attain a mean passenger waiting time above 20 minutes in steady state. In the simulation of the Self-Coordinating method, passenger waiting time increases to ten minutes, but the dispatching method then gradually controls big gaps. The proposed control method stabilizes the system within seven dispatches of the offset of the perturbation. Passenger waiting time is higher after the perturbation because more passengers were introduced into the system and operations were slowed down. The method, however, maintains the minimal headway variance at the maximal frequency and the lowest mean passenger waiting time.

CHAPTER V

CONCLUSION

In this work, the problem of dispatching buses on a high frequency route was addressed. The research objective was to find a control mechanism to send buses on a loop-shaped route with minimal headway variance. In Chapter 3, the bus dispatching problem was modeled as a finite state stochastic decision process. An optimal dispatching policy was derived into its analytical form by backward induction. This policy uses real-time information and prediction on the trajectory of every running bus at the time of control. In Chapter 4, the proposed dispatching policy was compared with a real-time control method recommended in the literature and the headway-based dispatching policy used in practice. The proposed policy was found to yield lower passenger waiting time on a wide range of operating conditions, and to be more resilient to perturbation.

The proposed dispatching method externalizes completely the modeling of headway and load dynamics. The joint probability distribution for the random variables K_i 's summarizes all the information required to dispatch buses with minimal headway variance for maximal frequency. Capacity constraints and the time of waiting passengers are not explicitly considered. The method takes an agency perspective to the problem and considers that if buses are dispatched with minimum headway variance at minimal mean headway, then the flux of arriving passengers will be evenly spread among the passing vehicles. Since headway and load variance increases monotonically along a route[20], maintaining the lowest possible headway variance assures that system performance is maximized.

In the derivation of Chapter 3, the probability distribution for the inter-arrival

times of buses on the route were left in implicit terms. As surveyed in the literature review, there exists prediction tools to generate unbiased joint probability functions. These tools can be used to feed the control policy. The field of real-time bus travel time prediction is evolving. As prediction tools improve in quality, the proposed control method will remain the optimal way of dispatching vehicle at a single control point. Future research includes the the generalization of the method for a loop-shaped route with several control point and the development of an implementation platform.

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